Problem 1.- (15 marks)
Solve the following systems of linear equations using the method of your choice: either the Gauss-Jordan method or the Cramer's rule (only if applicable). For each consistent system, don't forget to tell how many solutions you found? Whenever you find infinitely many solutions give also a particular solution.

(i)
$$\begin{aligned} 3x - 7y &= 0 \\ 6x - 14y &= 12 \end{aligned}$$

(ii)
$$\begin{aligned} x - 2y + 3z &= 7 \\ 2x - 3y &= 5 \\ x - 3y + 2z &= -5 \end{aligned}$$

(iii) $x_1 - x_3 + 2x_4 = 0$

$2x_1 + x_2 - x_3 - x_4 = 2$

$4x_1 + x_2 - 3x_3 + 3x_4 = 2$

Problem 2.-   (12 marks)

Evaluate the determinants of the following matrices and conclude about their **invertility:**

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 4 & 0 & 0 & 0 \\ 2 & -1 & 5 & 4 \\ 5 & 2 & 0 & -1 \end{bmatrix}$$

Problem 3.-   (12 marks).-

Invert the following matrix A using both well known methods:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 2 \\ 4 & 4 & 2 \end{bmatrix}$$

a) Using elementary row-operations.

b) Using cofactors and the adjoint of A.

Continuation of Problem 3

c) Use the inverse found previously in problem 3 to s

Problem 4.-   (10 marks)

Given the following matrices A, B and C

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 7 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 1 \\ 5 & 1 \\ 7 & 1 \end{bmatrix}$$

Calculate the following expressions (if possible):
5A , A^2 , A^T, AB, AC, A^2C^2 , $(AC)^2$, A + C, B^TB, adj(A) .

Problem 5.- (15 marks)

Here A, B and C represent square matrices of same size.

a) Simplify the following expressions:

(i) $(5\,B^{-1}B^{10}\,C^{-1}\,A\,)^{-1}$ (ii) $(5\,B^{-1}\;(B^{T})^{2}\,A^{T}\,)^{T}$

b) Expand the following expression: $(A + B)(\,I + A^{-1})(I + B^{-1})$ where I is the identity matrix of same size as A and B.

c) If A and B are 4 x 4- matrices with det(A) = -5 and det(B) = 10 , evaluate the following determinants:

(i) det(BA) =

(ii) $\det(A^{-1}\,B^{2}\,A)$,

(iii) $\det(2B^{-1}A)$

(iv) $\det((10B^2)^T)$

Problem 6.- (12 marks)

Let A, B and C be three points in the space $\mathbf{R}^3$ described by:
A = (5,1,-2)   B = (2,1,2)   and C = (9,1,1) .

a) Show that the triangle ABC is a right triangle. Specify where is the right angle.
b)

Continuation of Problem 6

Problem 7.- (12 marks)

a) Let M be the point **(1, -2, 5)** and **P** be the plane described by the equation :

2 x - y + 4z = 10 . Find the distance between M and the plane **P** .

b) Show that the line **L** described by the parametric equations:

$X = 2 – t$

$Y = 3t$

$Z = 3 - 10t$

Intersects the previous plane **P** at a point Q. Find the coordinates of Q.

c) Show that the new line **L'** described by the parametric equations:

$X = u$

$Y = 1 - u$

$Z = 3 + 2u$

Intersects the previous line **L** at a point R. Find the coordinates of R.

Problem 8.- (12 marks)

Here is the diagram of the traffic network around

Continuation of Problem 8