Dawson College, Mathematics Department
Final Examination

Linear Algebra 201-NYC-05 (Commerce)
(Sections 6, 7)

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1. (6 marks) Solve the linear system by Gauss-Jordan method.

$$x_1 \quad x_2 \quad 3x_3 \quad 4x_4 \quad 12$$
$$2x_1 \quad x_2 \quad 5x_3 \quad 7x_4 \quad 19$$
$$x_1 \quad x_2 \quad 2x_3 \quad 2x_4 \quad 5$$

Ans. $x_1 \quad 1 \quad 3t,\ x_2 \quad 2 \quad t,\ x_3 \quad 3,\ x_4 \quad t$

2. (6 marks) For which values of k the system has
   a) Exactly one solution, b) No solution, c) Infinitely many solutions

$$x \quad 2y \quad z \quad 3$$
$$2x \quad 5y \quad 3z \quad 8$$
$$x \quad 4y \quad k \quad 3\ z \quad k \quad 8$$

Ans. a) $k \quad 0$ b) 0 c) never

3. (5 marks) Let A be an invertible matrix, such that $2A^3 \quad 3A^2 \quad I \quad 2A$. Show that $A^{\ 1} \quad 2A^2 \quad 3A \quad 2I$

Ans. $AA^{\ 1} \quad A(2A^2 \quad 3A \quad 2I) \quad 2A^3 \quad 3A^2 \quad 2A \quad I$
$A^{\ 1}A \quad (2A^2 \quad 3A \quad 2I)A \quad 2A^3 \quad 3A^2 \quad 2A \quad I$

4. The following linear system is given.

$$\begin{matrix} x & 2y & z & 1 \\ 2x & y & z & 1 \\ x & y & z & 2 \end{matrix}$$

a) (5 marks) Find the inverse of the coefficient matrix
b) (3 marks) Use the result found in part a) to solve the system

Ans. a) $\begin{matrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{matrix}$ b) $x \quad 1,\ y \quad 3,\ z \quad 6$

5. (5 marks) Use the Cramer's rule to solve the following system for only.

$$\begin{matrix} x & 2y & z & 1 \\ 2x & y & z & 1 \\ x & y & z & 2 \end{matrix}$$

Ans. $y \quad 3$

6. (5 marks) Find the matrix X, if $\begin{matrix} 1 & 2 \\ 3 & 5 \end{matrix} X \quad \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{matrix} \begin{matrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{matrix}$

Ans. $\begin{matrix} 12 & 3 \\ 7 & 2 \end{matrix}$

7. Let A *and* B be $2 \quad 2$ matrix, such that $\det A \quad 2$, $\det B \quad 3$. Find

a) (4 marks) $\det A^{1} B^{T}$

b) (4 marks) $\det 2 \; |A| B^{1}$

c) (4 marks) $\det \; B\,adj\;3A$

d) (5 marks) Let $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \; 5$. Find $\begin{vmatrix} d & g & a \\ 3e \quad d & 3h \quad g & 3b \quad a \\ 2f \quad d & 2i \quad g & 2c \quad a \end{vmatrix}$

Ans. a) $\frac{3}{2}$ b) $\frac{1}{3}$ c) 54 d) 30

8. (3 marks) Let $\breve{u}$ *and* $\breve{v}$ be two vectors, such that $\breve{u} \quad \breve{v} \quad 1,2,3$. Find the vector

$\breve{u} \quad \breve{v} \quad 2\breve{v} \quad \breve{u}$ Ans. $3,6,9$

9. (3 marks) Let $\breve{u}$ $k, 1 \quad k, 2$, $\breve{v}$ $2, 1, k$. Find the values of k for which $\breve{u}$ $\breve{u}$ $\breve{v}$ 4 .

Ans. $k \quad 0, \; k \quad 3/2$

10. Let $\breve{u}$ $1,1,0$ and $\breve{v}$ $1,0,2$

a) (3 marks) Find $Proj_{\breve{u}} \; 2\breve{u} \quad \breve{v}$

b) (3 marks) Find the area of the triangle determined by $\breve{u}$ and $\breve{v}$

Ans. a) $3/2, 3/2, 0$ b) $3/2$

11. (4 marks) Find the equation of the plane which is perpendicular to the plane $x \quad y \quad 2z \quad 3$ and passes through the points $A\,1,0,2$ and $B\;0,1, \quad 1$.

Ans. $x \quad y \quad 1 \quad 0$

12. (4 marks) Find the point on the line $x \quad 6 \quad t, \; y \quad 2 \quad t, \; z \quad 3 \quad t$ which is the closest to the point $B\;0,1,1$

Ans. $x \quad 3, \; y \quad 1, \; z \quad 0$

13) (4 marks) Find the distance from the point $A\,1,0,1$ to the line $x \quad 1 \quad t, \quad y \quad 2 \quad t,$ $z \quad 3 \quad t$

Ans. $\frac{2}{3}\sqrt{6}$

14. (4 marks) Find the distance between the lines $\begin{matrix} x & 1 & 2t \\ y & 2 & t \\ z & 3 & t \end{matrix}$ and $\begin{matrix} x & 1 & s \\ y & & s \\ z & 1 & s \end{matrix}$

Ans. ˎ

15. (10 marks) Maximize $P \quad 4x_1 \quad 3x_2 \quad 2x_3$ subject to $\begin{matrix} 2x_1 & 5x_2 & x_3 & 40 \\ 2x_1 & 2x_2 & x_3 & 15 \\ x_1 & 2x_2 & x_3 & 2 \end{matrix}$

Ans. Max $P \quad 30$ at $13, 0, 11$

16. (10 marks) Minimize $_3$ subject to $\begin{matrix} x_1 & 2x_2 & 2x_3 & 5 \\ 2x_1 & & x_3 & 4 \\ 3x_1 & x_2 & x_3 & 2 \end{matrix}$

Ans. Min $C \quad 40$ at $5, 0, 0$