

**Question 1.** If the limit exists, find its value; otherwise explain why the limit doesn't exist.

a. (3 marks)

$$\lim_{x \rightarrow 9} \frac{x^3 - 27}{x - 3}$$

**Answer:** DNE

b. (3 marks)

$$\lim_{x \rightarrow \infty} \frac{3x^3 - x + 1}{5x^3}$$

**Answer:**  $\frac{3}{5}$

c. (3 marks)

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^4 + 2x^3}$$

**Answer:**  $\frac{1}{2}$

**Question 2.** (5 marks) For which values of  $x$  is the following function continuous? Clearly explain your reasoning.

$$f(x) = \begin{cases} \frac{3x-2}{x^2-1} & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ x^4 + x^3 + x + 2 & \text{if } x > 2 \end{cases}$$

**Answer:**  $\mathbb{R} \setminus \{2\}$

**Question 3.** Given

$$f(x) = x^2 - 2x + 1$$

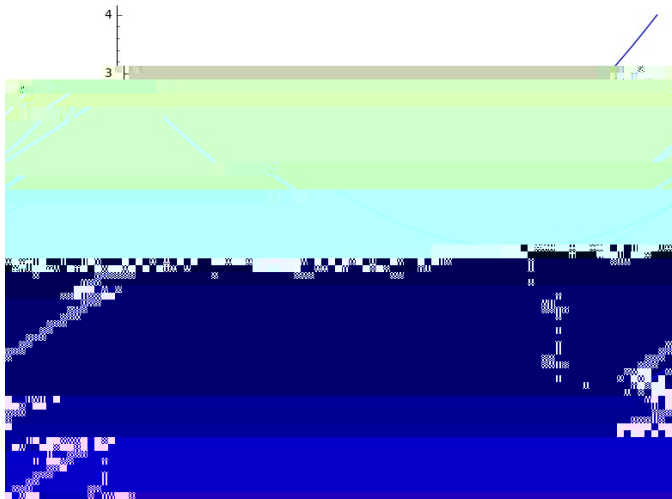
a. (4 marks) Find the derivative of  $f(x)$  using the definition of the derivative as a limit.

**Answer:**  $f'(x) = 2x - 2$

b. (3 marks) Find the tangent to  $f(x)$  at  $x = 2$ .

**Answer:**  $y = 2x - 3$

c. (3 marks) Sketch the graph of  $f(x)$  and its tangent at  $x = 2$ .



**Question 4.** (4 marks) Find the absolute maximum value and the absolute minimum value of the given function

$$f(x) = \frac{x}{x^2 + 1} \quad \text{on } [ -1; 1]:$$

$$\text{Answer: abs. min. } f(-1) = \frac{1}{2}, \text{ abs. max. } f(1) = \frac{1}{2}$$

**Question 5.** Find the derivative of the following functions:

a. (5 marks)

$$f(x) = \arcsin e^{2x}$$

$$\text{Answer: } f'(x) = \frac{2e^{2x}}{1 - e^{4x}}$$

b. (5 marks)

$$f(x) = \frac{(\ln x + x^2)^2}{\sin x}$$

$$\text{Answer: } f'(x) = \frac{(\ln x + x^2) \left[ \left( \frac{2}{x} + 4x \right) \sin x - (\ln x + x^2) \cos x \right]}{\sin^2 x}$$

c. (5 marks)

$$f(x) = (x^2 + 1) \arctan x$$

$$\text{Answer: } f'(x) = 1 + 2x \arctan x$$

**Question 6.** (5 marks) Using logarithmic differentiation, find the derivative of the function

$$f(x) = (x^3 + 1)^{12} \sqrt[3]{2x^2 + 5x} (x \tan x)^3.$$

Do not simplify (expand) your answer.

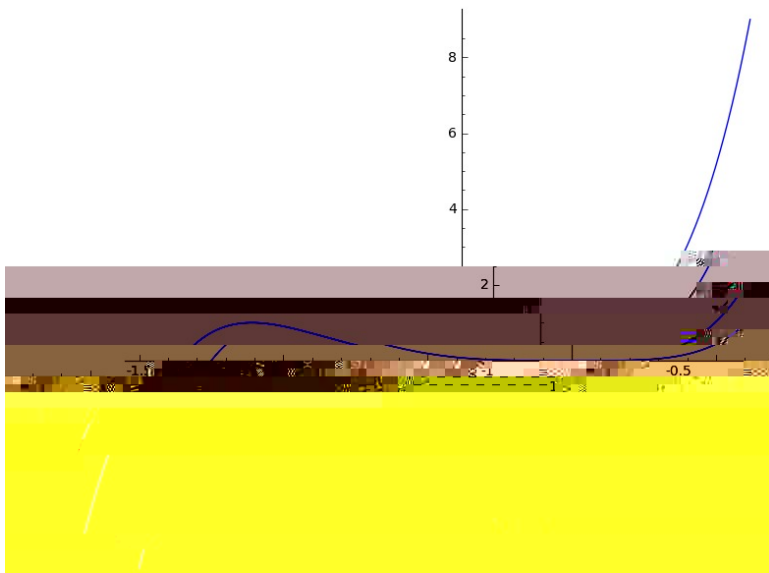
$$\text{Answer: } f'(x) = (x^3 + 1)^{12} \sqrt[3]{2x^2 + 5x} (x \tan x)^3 \left( \frac{36x^2}{x^3 + 1} + \frac{4x + 5}{6x^2 + 15x} + \frac{3}{x} + \frac{2 \sec^3 x}{\tan x} \right)$$

**Question 7.** (5 marks) Find an equation of the tangent line to the curve  $x^2 y^3 - y^2 + xy - 1 = 0$  at the point (1;1).

$$\text{Answer: } y = \frac{3}{2}x + \frac{5}{2}$$

**Question 8.** (5 marks)

a. (3 marks) Show that the demand equation is  $p = 0.0000002x$



**Question 12.** (4 marks) Find the horizontal asymptote(s) and vertical asymptote(s) (if any) of the following function.

$$f(x) = \frac{x^2 - 2x}{x^2 - 1}$$

**Answer:**  $x = 1; x = -1; y = 1$

**Question 13.** (4 marks) Evaluate the following integral.

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