(b) $\int x(x^2 + 3)^4\, dx$

16. **(16 marks)** Consider the function $f(x) = \dfrac{x^3 \quad 1}{x^3 + 1}$. The rst and second derivatives of f are

$$f'(x) = \frac{6x^2}{(x^3 + 1)^2} \qquad \text{and} \qquad f''(x) = \frac{12x(1 \quad 2x^3)}{(x^3 + 1)^3}.$$

(a) Give the domain of f.
(b) Find the intercepts.
(c) Find the horizontal and/or vertical asymptotes (if any).
(d) Find the intervals where f is increasing, and the intervals where f is decreasing.
(e) Find the local maxima and local minima.
(f) Find the intervals where f is concave up, and the intervals where f is concave down. Give the in ection point(s).
(g) Use the information above to sketch the graph of f. Clearly label all the important points on the graph.

Answers:

1. (a) 1=4 (b) 2 (c) 3 (d) 1
2. 4
3. $\dfrac{1}{(x + 1)^2}$
4. $f'(x) = \dfrac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$
5. $x = 2$ and $x = \quad 1=2$
6. $f''(x) = 4 \quad 4\sin(2x)$
7. $\dfrac{\sin x \quad 3y^2 \quad \sec^2 x}{6xy \quad 1}$
8. 3=5
9. $y' = x^{\sin x}(\cos x \ln x + \dfrac{\sin x}{x})$
10. $y = \quad 2x + 3$
11. $\dfrac{2}{(x^2 + 1)^{3=2}}$
12. 10:4 feet per second
13. 4000 cm^3
14. Absolute maximum value is $\quad 1$, absolute minimum value is $\quad 5$.
15. (a) $\dfrac{2x^{2=3}}{3} \quad \dfrac{2x^{1=2}}{+} \dfrac{x^3}{3} + C$ (b) $\dfrac{(x^2 + 3)^5}{10} + C$
16. (a) $\mathbb{R} \setminus f \quad 1g$ (b) x-intercept $(1; 0)$, y-intercept $(0; \quad 1)$ (c)vertical asymptote: $x = \quad 1$ 3. 40005000