**Name:** ________________________________________

**Student ID:** ________________________________________

# WINTER 2012 FINAL EXAM

## Calculus for Electronics Engineering Technology

**Dawson College: Department of Mathematics**

**Date: May 22nd 2012, 9:30am to 12:30pm**

**Course Code: 201-NYA-05 Section 6**

**Examiner: Emilie Richer**

**INSTRUCTIONS:**

- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer use the back of the page.
- SHOW ALL YOUR WORK: Show all your work clearly and justify all your answers.
- Verify that your final examination copy has a total of 19 pages including the cover page.

| Question | # Marks | |
|---|---|---|
| 1 | 10 | |
| 2 | 6 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 5 | |
| 6 | 5 | |
| 7 | 5 | |
| 8 | 5 | |
| 9 | 5 | |
| 10 | 12 | |
| 11 | 10 | |
| 12 | 12 | |
| 13 | 5 | |
| 14 | 5 | |
| 15 | 5 | |

**Question 1.** *(10 marks (1 mark each))*

**Question 2.** *(6 marks (1 mark each))*

Integrate the following.

(a)

$$\int 4x^3 - \sqrt{x}\, dx$$

(b)

$$\int (2x^2 - 3)^2\, dx$$

(c)

$$\int 2x^3 + \cos x\, dt$$

(d)

$$\int e^x - \frac{1}{x}\, dx$$

(e)

$$\int \frac{2}{x^3} + e^\pi\, dx$$

(f)

$$\int \frac{41x^3 - 3x^2 + 1}{x}\, dx$$

**Question 3.** *(5 marks)*

Sketch a graph that satisfies all of the following conditions:

$\lim_{x \to \infty} f(x) \to \infty$

$\lim_{x \to 1^+} f(x) \to \infty$

$\lim_{x \to 1^-} f(x) \to -\infty$

$\lim_{x \to -1} f(x) = -1$

$f(2) = 0$

$\lim_{x \to 2} f(x)$ does not exist

$f(0) = 3$

**Question 4.** *(5 marks)*
Evaluate the following limits. If the limit does not exist, determine if its one-sided limits tend to $\pm\infty$.

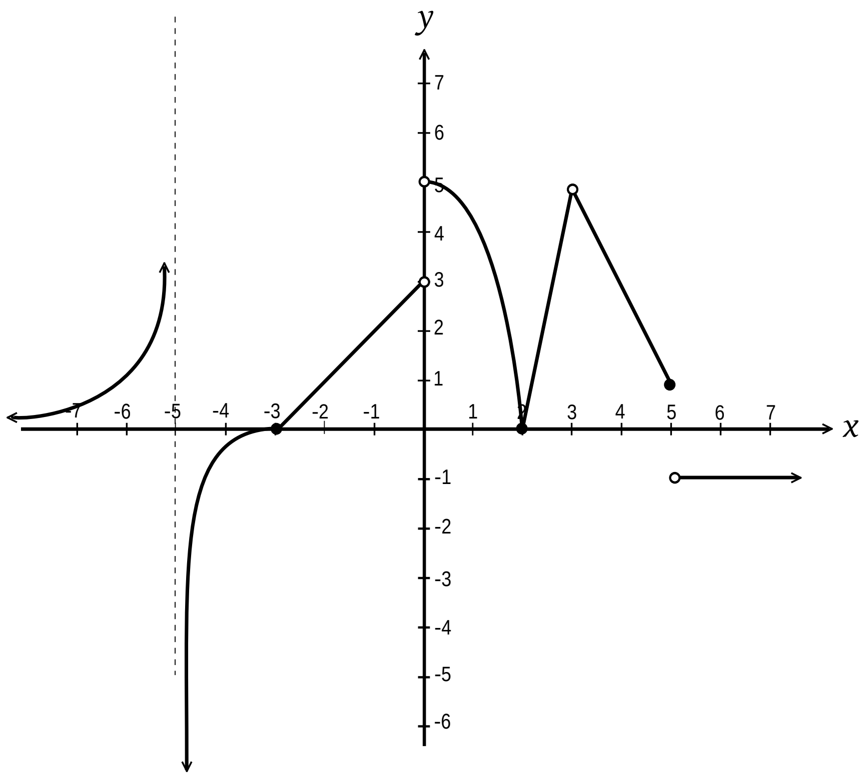
(a) $\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$

(b) $\lim_{x \to -1} \frac{x^2+3x+2}{x^2-1}$

(c) $\lim_{x \to -\infty} \frac{x^2+3x^4-7x}{2x^3+2}$

(d) $\lim_{x \to \infty} \frac{3x^7-x^8+2}{3x^8-7}$

**Question 5.** *(5 marks)*
Use the graph of $y = f(x)$ pictured above to find the following values. If the value does not exist, write *DNE*.




(a) $f(0)$ = ________

(b) $\lim_{x \to 5^+} f(x)$ = ________

(c) $\lim_{x \to -5^+} f(x)$ = ________

(d) $\lim_{x \to 4} f(x)$ = ________

(e) $\lim_{x \to +\infty} f(x)$ = ________

(f) $\lim_{x \to 5} f(x)$ = ________

(g) $\lim_{x \to -3^-} f(x)$ = ________

(h) $\lim_{x \to -\infty} f(x)$ = ________

(i) $f(5)$ = ________

(j) $\lim_{x \to 0^-} f(x)$ = ________

(k) $f'(-1)$ = ________

(l) $\int_{-3}^{0} f(x)\,dx$ = ________

(m) $\int_{5}^{7} f(x)\,dx$ = ________

**Question 6.** *(5 marks)*
Sketch the curves $y = 2\cos x$, $y = 1$ and find the area between them for $0 \leq x \leq \pi$.

**Question 7.***(5 marks)*
Use logarithmic differentiation to find the derivative of the function $y = (\cos x)^{2x}$

**Question 8.** *(5 marks)*
Find the value of the constant $a$ if the slope of the tangent line to the curve $y = -6ax^2 + 6x + 4$ at $x = -2$ is equal to 3.

**Question 9.** *(5 marks)*

Find the equation of the tangent line to the curve $f(x) = e^{2x} - 3x$ at the point $(0, 1)$.

**Question 10.** *(12 marks (3 marks each))*
Find the derivatives of the following functions.

(a) $h(t) = e^{\cos(4t)}$

(b) $g(z) = 3z^{-2}\ln(\sin z)$

(c) $f(x) = \log_3(\tan(x^3))$

(d) $g(x) = (2x - 1)(\sin(4x))(e^{-x}$

**Question 11.** *(10 marks)*
Sketch the graph of $f(x) = x^3 - 3x$. Find and clearly identify on the sketch the following:

(a) The $x$ and

(d) The intervals where $f(x)$ is concave up/down and any points of inflection

SKETCH OF $f(x) = x^3 - 3x$

**Question 12.** *12 marks (3 marks each)*
Integrate the following.
(a)

$$\int \frac{-2\sin(2x)}{\cos 2x}\,dx$$

(b)

$$\int (20x^4 - 18x^2)(2x^5 - 3x^3)^{-8}\,dx$$

(c)

$$\int \sin^3 x \cos^2 x \, dx$$

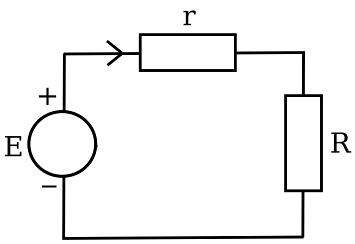
(d)

$$\int_{-1}^{4} x\sqrt{8-x}\, dx$$

**Question 13.** *(5 marks)*
A discharged ($V_c = 0$ at $t = 0$) 4mF capacitor is to be charged by a current of $i = 25e^{1-0.75t}$mA. Find the capacitor voltage ($V_c$) at $t = 135$ms.

**Question 14.** *(5 marks)*

In the electric circuit shown below, the voltage $E = 5$ (in volts) and resistance $r = 100$ (in ohms) are constant, $R$ is the resistance of a load.




In such a circuit, the electric current $i$ is given by $\frac{E}{r+R}$ and the power $P$ delivered to the load $R$ is given by $P = Ri^2$.

Given that $R$ is positive, determine the value of $R$ so that the power $P$ delivered to $R$ is a maximum.

**Question 15.** *(5 marks)*
Use implicit differentiation to find the $y'$ in the following equations.
(a) $x^2y^3 + x + 2y = 0$

(b) $\ln(x \sin y) + y = x^2$

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**Name:** ________________________________________