DAWSON COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

CALCULUS-III (201-BZF-05)

Ma 24, 2012 **T e: 14:00-17:00 . .**

I c W:R. Fournier and T. Kengatharam

Na e:
ID:

I c W :

- Translation and regular dictionaries are permitted.
- Scientific non-programmable calculators are permitted.
- Print your name and ID in the provided space.
- This examination booklet must be returned intact.

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(3) [*5 marks*]Find a power series expansion for $f(x) = \frac{x}{(x+2)^2}$. (You may use $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.)

(4) [*5 marks*]Given that $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, find a power series expansion for $\cos\left(x + \frac{\pi}{4}\right)$.

(5) [*5 marks*]Find the equation of the tangent line to the curve $\underline{r}(t) = (\cos t; \sin t; t)$ at the point $(-1; 0; \pi)$.

(6) [*5 marks*]Find the point(s) on the curve with equation $\underline{r}(t) = (\cos t; \sin t; t)$ at which the curvature $\kappa = \frac{|\underline{r}' \times \underline{r}''|}{|\underline{r}'|^3}$ is maximal.

(7) [$5\ marks$]Find the arc-length parametrization for the curve $\underline{r}(t) = (\cos t; \sin t; t)$, $t \geq 0$.

(8) [$5\ marks$]The binormal $\underline{B}(t)$ is defined as $\underline{B}(t) = \underline{T}(t) \times \underline{N}(t)$, where $\underline{T}(t)$ is the unit tangent vector and $\underline{N}(t)$ is the unit normal vector of a smooth curve C at any point $\underline{r}(t) \in C$. Prove that $\underline{B}(t)$ and $\underline{B}'(t)$ are perpendicular.

(9) [*5 marks*]Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{xy\sin(xy)}{x^2+y^2}:$$

(12)

(14) [*5 marks*]Find all critical points of $f(x;y) = xy - x^2 - y^2$ and classify them.

(15) [*5 marks*]Prove that $f(x; y) = xe^x \cos y - ye^x \sin y$ is a solution of the partial differential equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

(16) [*5 marks*]Compute the integral

$$\int\int_R \frac{y + xy}{1 + y^2} dA$$

where R is the rectangle $[0; 2] \times [0; 1]$.

(17) [$5\ marks$]Compute the integral

$$\int\int_R xydA$$

where D is the disc $\{(x;y) \mid x^2 + y^2 \leq 1\}$.

(18) [$5\ marks$]Compute the volume of the tetrahedron bounded by the plane $x + y + z = 1$ and the three coordinate planes.

(19) [*5 marks*]Use polar coordinates to compute the volume of the region lying below the cone with equation $z = \sqrt{x^2 + y^2}$ and above the disc with equation $x^2 + y^2 \leq 1$.

(20) [*5 marks*]Evaluate

$$\int\int\int_E xe^{(x^2+y^2+z^2)^2} dV$$

where E is the upper hemisphere

$$\{(x; y; z) \mid x^2 + y^2 + z^2 \leq 1; \ z \geq 0\}:$$

(You may use the spherical polar coordinates: $x = \rho \sin\phi \cos\theta; \ y = \rho \sin\phi \sin\theta; \ z = \rho \cos\phi; \ dV = \rho^2 \sin\phi d\rho d\phi d\theta$).