DAWSON COLLEGE

Mathematics Department

Final Examination

Engineering Mathematics II (201-942-DW)

May 16, 2011

Instructor: N. Sabetghadam

Time: 3 Hours

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Instructions:

- Print your name and ID in the provided space.
- Solve the problems in the space provided for each question and show all your work clearly.
- A Formula sheet is attached.
- Scienti c non-programmable calculators are permitted.
- This examination booklet must be returned intact.

This examination consists of 12 questions. Please ensure that you have a complete examination booklet before starting.

1.(5 marks) Evaluate the following limit.

$$\lim_{x \to -1} \frac{x^2 \quad 1}{3x + 3}$$

Solution: $\lim_{x \to -1} \frac{x^2 \quad 1}{3x + 3} = \lim_{x \to -1} \frac{(x \quad 1)(x + 1)}{3(x + 1)} = \lim_{x \to -1} \frac{x \quad 1}{3} = \frac{\quad 2}{3}$

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2.(5 marks) Find the derivative of the following function by using the \de nition of derivative".

$$f(x) = 8x^2 \quad 5x + 1$$

Solution: $\lim_{h \to 0} \frac{f(x + h) \quad f(x)}{h} = \lim_{h \to 0} \frac{8(x + h)^2 \quad 5(x + h) + 1 \quad (8x^2 \quad 5x + 1)}{h} =$
$\lim_{h \to 0} \frac{8x^2 + 16xh + 8h^2 \quad 5x \quad 5h + 1 \quad 8x^2 + 5x \quad 1}{h} = \lim_{h \to 0} \frac{16xh + 8h^2 \quad 5h}{h} = \lim_{h \to 0} \frac{h(16x + 8h \quad 5)}{h} =$
$\lim_{h \to 0} 16x + 8h \quad 5 = 16x \quad 5$

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3.(5 marks) Find the \second derivative" of the given function.

$$f(r) = r(2r + 1)^{3}h$$

6.(5 marks) Find the equations of the tangent and the normal lines to the indicated curve at the point $(1, 4)$.

$$y = 6x - 2x^2$$

Solution: $y' = 6 - 4x \rightarrow m_{tan} = 6 - 4(1) = 2 \rightarrow m_{nor} = -\frac{1}{2}$

So the tangent line at point $(1, 4)$ is $y - 4 = 2(x - 1) \rightarrow y = 2x + 2$ and the normal line is

$$y - 4 = -\frac{1}{2}(x - 1) \rightarrow y = -\frac{1}{2}x + \frac{9}{2}.$$

7.(5 marks) A rectangular garden is to be enclosed with 8100 meter of fencing. Find the maximum possible area of the garden.

Solution: Let x and y be the length and the width of the garden respectively. So $2x+2y = 8100 \rightarrow y = 4050 - x$. The area A of the garden is $A = xy = x(4050 - x) = 4050x - x^2$. To find the maximum possible area, we should take derivative of the area in terms of x. Therefore $A' = 4050 - 2x = 0 \rightarrow x = \frac{4050}{2} = 2025$. If $x = 2025$ then $y = 4050 - 2025 = 2025$ and the maximum area would be $(2025)^2 = 4100625$.

8.(5 marks) Calculate Δy and dy for the given values of x and Δx.

$$y = x\sqrt{1 + 4x} \qquad x = 12 \qquad \Delta x = 0.06$$

Solution:

$$\Delta x = f(12.06) - f(12) = (12.06\sqrt{1 + 4(12.06)}) - (12\sqrt{1 + 4(12)}) = 84.6265 - 84 = 0.6265$$

$$y' = \sqrt{1 + 4x} + 2x(1 + 4x)^{-\frac{1}{2}} \rightarrow dy = y'dx = (\sqrt{1 + 4(12)} + 2(12)(1 + 4(12))^{-\frac{1}{2}})(0.06) = 0.6257$$

9.(20 marks)

10.(25 marks) Evaluate the following integrals:

(a) $\int (\frac{x^2}{2} + \frac{2}{x^2})dx$

Solution: $\int (\frac{x^2}{2} + \frac{2}{x^2})dx = \int (\frac{x^2}{2} + 2x^{-2})dx = \frac{x^3}{6} \quad 2x^{-1} + C$

(b) $\int x^3(x^4 + 1)^4 dx$

Solution: $\int x^3(x^4 + 1)^4 dx = \frac{1}{4}\int 4x^3(x^4 + 1)^4 dx = \frac{(x^4 + 1)^5}{20} + C$

(c) $\int \sin^5 x \cos x dx$

Solution: $\int \sin^5 x \cos x dx = \frac{\sin^6 x}{6} + C$

(d) $\int xe^{-x^2} dx$

Solution: $\int xe^{-x^2} dx = \frac{1}{2}\int 2xe^{-x^2} dx = \frac{1}{2}e^{-x^2} + C$

(e) $\int_2^5 (\frac{1}{x\sqrt{x}} + 4)dx$

Solution: $\int_2^5 (\frac{1}{x\sqrt{x}} + 4)dx = \int_2^5 (x^{-3/2} + 4)dx = \frac{x^{-1/2}}{1/2} + 4x |_2^5 =$