

**POLYNOMIALS, DIVISION AND FACTORIZATION****LONG DIVISION OF POLYNOMIALS**

Example 1: Divide the polynomial  $f(x) = 2x^3 + 3x^2 - 2x + 1$  by  $g(x) = x^2 + 2x + 1$ .

$$\begin{array}{r} 2x - 1 \\ \hline x^2 + 2x + 1 \overline{)2x^3 + 3x^2 - 2x + 1} \\ 2x^3 + 4x^2 + 2x \\ \hline -x^2 - 2x + 1 \\ -x^2 - 2x - 1 \\ \hline 0 \end{array}$$

Step 1: Divide the first term of the dividend by the first term of the divisor.

Step 2: Multiply the result by the divisor and subtract from the dividend.

Step 3: Repeat the process until the degree of the remainder is less than the degree of the divisor.

Step 4: The quotient is  $2x - 1$  and the remainder is 0.

We see that the polynomial has been divided completely.

Step 5: The final answer is  $2x - 1$ .

Step 6: The remainder is 0.

Step 7: The quotient is  $2x - 1$ .

Step 8: The final answer is  $2x - 1$ .

Step 9: The remainder is 0.

Step 10: The quotient is  $2x - 1$ .

Step 11: The final answer is  $2x - 1$ .

Step 12: The remainder is 0.

Step 13: The quotient is  $2x - 1$ .

Step 14: The final answer is  $2x - 1$ .

Step 15: The remainder is 0.

Step 16: The quotient is  $2x - 1$ .

Step 17: The final answer is  $2x - 1$ .

Step 18: The remainder is 0.

Step 19: The quotient is  $2x - 1$ .

Step 20: The final answer is  $2x - 1$ .

Step 21: The remainder is 0.

Step 22: The quotient is  $2x - 1$ .

Step 23: The final answer is  $2x - 1$ .

Example: Use long **vision** to find the **treasure**



Example 2:  $n(x) = x^3 - 3x^2 - x + 3$ .

The possible rational zeros are

$$\frac{p}{q} \text{ where } p = \pm 1, 3 \quad q = \pm 1, 3$$

The possible rational zeros are  $\pm 1, \pm 3, \pm \frac{1}{3}, \pm \frac{3}{1}$ .  
By dividing each by 3, we find that the possible rational zeros of  $n(x) = x^3 - 3x^2 - x + 3$  are  $\pm 1, \pm \frac{1}{3}$ .

$$\begin{array}{r} x^3 - 3x^2 - x + 3 \\ \underline{- (x-1)} \\ x^2 - 2x - 3 \\ \underline{- (x-3)} \\ x+1 \end{array}$$

$$\begin{array}{r} x^3 - 3x^2 - x + 3 \\ \underline{- (x+1)} \\ x^2 - 2x + 3 \\ \underline{- (x+1)} \\ x-3 \end{array}$$

The factor theorem tells us that  $n(x) = 0$  if and only if  $x = -1$  or  $x = 3$ .  
Thus, the only rational zero is  $x = 3$ , which corresponds to the root  $(3, 0)$ .

It may also be helpful to recall that if a polynomial has a rational zero, then it must have a rational factor.

With this in mind, we can write  $n(x) = 0$  as  
$$x^3 - 3x^2 - x + 3 = (x+1)(x-3)^2 = 0$$

From this, we see that the three roots are  $-1$  and  $3$  (each with multiplicity 2).

Finally, we can determine the behavior of the graph near the vertical asymptote at  $x = 3$  by examining the limits of  $n(x)$  as  $x$  approaches 3 from the left and from the right.

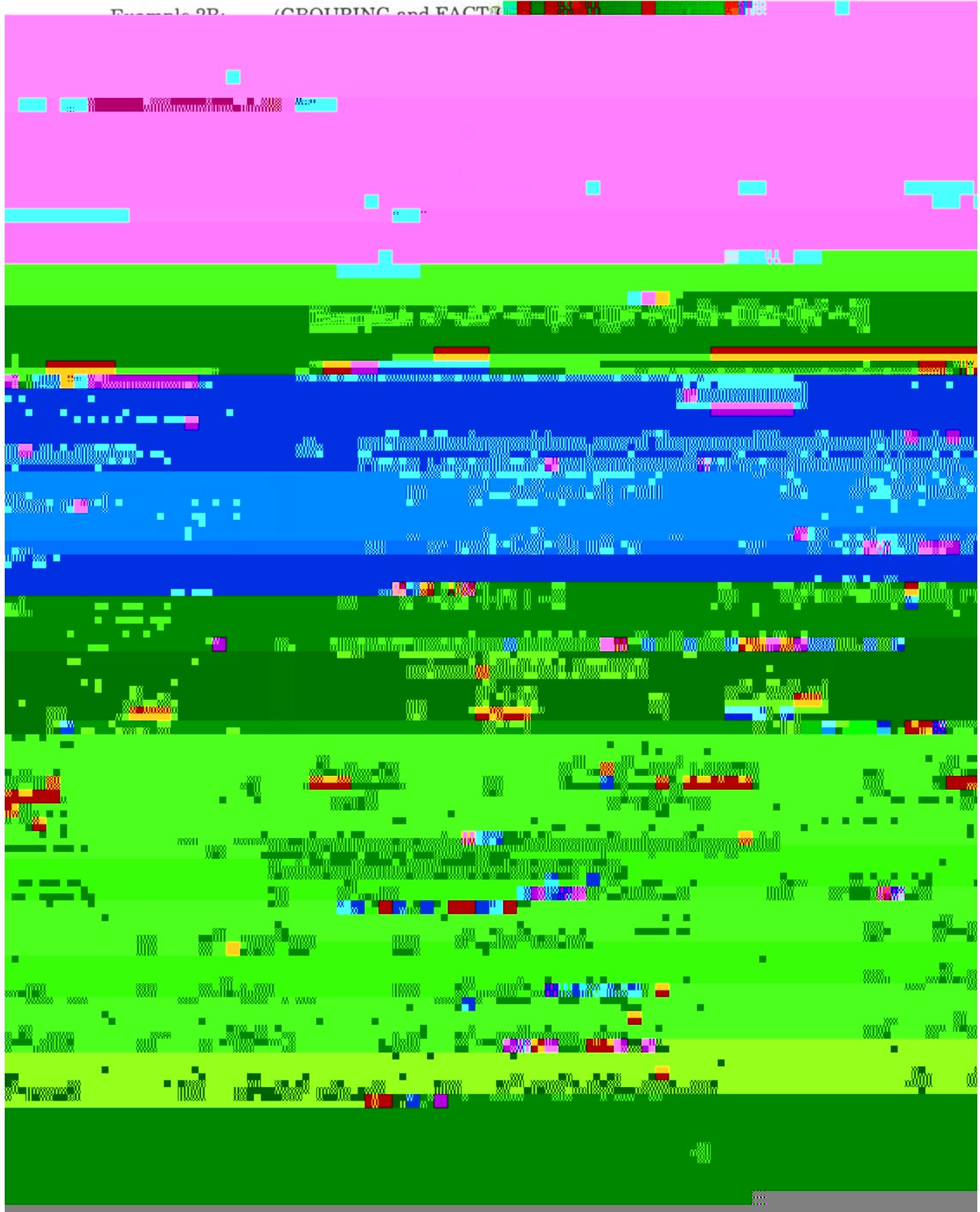
As  $x \rightarrow 3^-$ ,  $n(x) \rightarrow -\infty$  and as  $x \rightarrow 3^+$ ,  $n(x) \rightarrow \infty$ .

Therefore, the graph has a hole at  $(-1, 0)$  and a vertical asymptote at  $x = 3$ .

It may also be helpful to recall that if a polynomial has a rational zero, then it must have a rational factor.

Example 2B:

(GROUPING and FACTORING)



## ANSWERS

1)  $(x - 1)(x + 1)(x^2 + 1)^{1000} \cdot (-4)$  A.  $x(x + 1)(x - 2)(x - 8)$

2)  $(x^2 - 2x + 1)(x^2 + 4x + 4) = (x - 1)^2(x + 2)^2$  B.  $(x - 2)(x^2 + 4x + 4)$

3)  $(x + 2)^2(x + 4) \neq 6$  C.  $(x + 1)(x + 3)(x + 2)$

1)  $1 + \frac{4x + 1}{y^2 - 4x^2}$  D.  $x^2 - 2x + 7 - \frac{9}{x + 2}$  B.