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- There are a total of 100 marks
- There are a total of 12 pages
- Show all your work unless instructed otherwise. Do not write marks.
- You may use the back of the pages.
- Do not remove any pages from the book.

1. Find  $\int_{-4}^2 \left(-3 - \frac{3}{2}x\right)$

[4 pt]

(a) taking the limit

[2 pt]

(b) interpreting the

Summation formulas

$$\sum_{i=1}^n c = cn$$

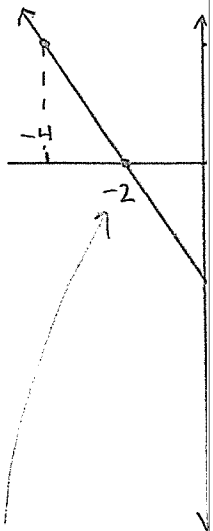
(a)  $\Delta x = \frac{b}{n}$ ,

$$\int_{-4}^2 \left(-3 - \frac{3}{2}x\right)$$

(b)  $f(x) = -3 -$

$$f(-4) = 3$$

$$f(2) = -6$$



$$-3 - \frac{3}{2}x = 0$$

$$\Rightarrow x = -2$$

2. Eval

[5 pt]

(a)

$\Rightarrow$

$\uparrow$

[5 pt]

(b)

[5 pt]

$$(c) \int \frac{1}{v(v+3)^2} dv = \int \left( \frac{A}{v} + \frac{B}{v+3} + \frac{C}{(v+3)^2} \right) dv$$

$$1 = A(v+3)^2 + B(v+3) + C$$

$$v=0 \Rightarrow 1 = 9A$$

$$v=-3 \Rightarrow 1 = -3C$$

$$v=-2 \Rightarrow 1 = A - C$$

$$\int \frac{1}{v(v+3)^2} dv = \int \left( \frac{1}{9v} + \frac{-1}{3(v+3)} + \frac{1}{(v+3)^2} \right) dv$$

$$= \frac{1}{9} \ln |v| - \frac{1}{3} \ln |v+3| - \frac{1}{v+3} + C$$

$$= \ln \left| \frac{v}{(v+3)^3} \right| + C$$

[5 pt]

$$(d) \int e^{\sqrt{3x+1}} dx$$

$$= \int e^t \cdot \frac{2}{3} t dt$$

$$= \frac{2}{3} \int t e^t dt$$

By parts  $\longrightarrow$

$$= \frac{2}{3} \left( t e^t - \int e^t dt \right)$$

$$= \frac{2}{3} t e^t - \frac{2}{3} e^t + C$$

$$= \frac{2}{3} \sqrt{3x+1} e^{\sqrt{3x+1}} - \frac{2}{3} e^{\sqrt{3x+1}} + C$$

[5 pt] 3. Fir

70

[5 pt] 4. Fir

71

72

73

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83

84

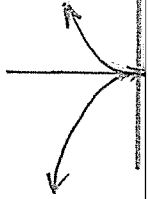
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87

[5 pt]

5. Deter



[5 pt]

6. Find th

5  
7

[8 pt.]

7. Let  $\mathcal{R}$  be

(a) Stat  
 $\mathcal{R}$  al

(b) Stat  
 $\mathcal{R}$  al

(a)

y = 1 -

(a)

(b)

↑  
2 - y  
↓

8. In all

[2 pt] (a)  $I$   
(

[2 pt] (b)  $I$

[1 pt] (c)  $I$

(

(

(

[3 pt] 9. Suppo

$$\int_0^0$$

$$\lim_{x \rightarrow 0^+}$$



] 10. Evaluate

$a$

$\lim_{n \rightarrow a}$

11. Suppose  
integrate

Dete

$f(x)$

$\therefore$

$=$

12. (a) Show that  $0 < \frac{1}{2}$   
 (b) Use part (a) (whether it converges or diverges. If it

$$(a) \frac{(n!)^2}{(2n)!} >$$

$$\frac{(n!)^2}{(2n)!} =$$

(b)

$$\therefore a_n$$

13. Determine if each of the

(a)  $\pi - \frac{2\pi}{e} + \frac{4\pi}{e^2} - \frac{8\pi}{e^3} + \dots$

Geometric

Since  $|r| < 1$

$$S = \frac{a}{1-r}$$

(b)  $\sum_{n=1}^{\infty} a_n$  where the

$$\sum_{n=1}^{\infty} a_n = L \quad n \rightarrow \infty$$

$\therefore$  the

14. Determine if each of the following series converges or diverges. Clearly state which test you use and your conclusion.

[4 pt]

$$(a) \sum_{n=2}^{\infty} \frac{1}{n \sqrt[3]{\ln n}}$$

Integral Test

$$\bullet f(x) \geq 0 ?$$

$$\bullet f(x) \text{ cts?}$$

$$\bullet f(x) \text{ dec?}$$

$$\int_2^{\infty} \frac{1}{x \sqrt[3]{\ln x}} dx =$$

=

=

=

$$\text{Since } \int_2^{\infty} \frac{1}{x \sqrt[3]{\ln x}}$$

[4 pt]

$$(b) \sum_{n=6}^{\infty} \left(1 - \frac{5}{n}\right)^{n^2}$$

Root test (no)

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left(1 - \frac{5}{n}\right)^{n^2}}$$

$$\Rightarrow \ln L = \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= \lim_{n \rightarrow \infty}$$

$$= 1$$

[4 pt]

$$(c) \sum_{n=2}^{\infty} (-1)^{n+1} \sqrt{\frac{1}{n^2-2}}$$

Alternating series

$$a_n = (-1)^{n+1} \frac{1}{\sqrt{n}}$$

$$\textcircled{1} b_n \rightarrow 0?$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2-2}}$$

$$\textcircled{2} b_n \text{ decreasing}$$

$$\therefore \sum a_n \text{ converges}$$

Now consider  $\sum$ 

$$\frac{1}{\sqrt{n^2-2}} >$$

Since  $\sum \frac{1}{n}$  diverges

Direct Comparison

Since  $\sum a_n$  converges $\sum a_n$  is conditional

[4 pt] 15. (a) Find the interval of convergence.

[4 pt] (b) Show that the Taylor series converges to  $f(x)$  on the interval of convergence.

[1 pt] (c) Use (a) and (b) to evaluate the series at  $x = \frac{5}{2}$  and  $x = \frac{1}{2}$ .

(a) Ratio Test:

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} (1)^{n+1}}{(-1)^n (1)^n} \right|$$

The series converges for  $|x - 2| < 1$ .

At  $x = 2 \Rightarrow$  converges

At  $x = 4 \Rightarrow$  diverges

Each of the endpoints is an interval of convergence.

(b)  $f^{(0)}(x) = (1-x)^{-1}$

$f^{(1)}(x) = -2(1-x)^{-2}$

$f^{(2)}(x) = 2(1-x)^{-3}$

$f^{(3)}(x) = -6(1-x)^{-4}$

$f^{(n)}(x) = (-1)^n n! (1-x)^{-(n+1)}$

$f^{(n)}(3) = (-1)^n n! (1-3)^{-(n+1)}$

$= (-1)^n n! (-2)^{-(n+1)}$

(c)  $f\left(\frac{5}{2}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n n! (-2)^{-(n+1)}}{n!}$

$\frac{1}{\left(\frac{5}{2} - 2\right)^2} = \sum_{n=0}^{\infty} \frac{(-1)^n n! (-2)^{-(n+1)}}{n!}$

$\frac{1}{\left(\frac{1}{2}\right)^2} = \sum_{n=0}^{\infty} \frac{(-1)^n n! (-2)^{-(n+1)}}{n!}$