DAWSON COLLEGE
Mathematics Department
Final Examination
201- !2-DW
Decem"er 1#th $ 2022

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Problem 1.(4+4+3 marks) Evaluate the limits:

$$a)\quad \lim_{x \to 5} \frac{x^2 - 25}{x^2 - 7x + 10}$$

$$b)\quad \lim_{x \to \infty} \frac{3x^4 - x^3 - 2x^2 + 5}{-5x^4 - 7x^2 + 11}$$

c) $\displaystyle \lim_{x \to \infty} \frac{3x^4 + 2}{x + 1}$

Problem 2. (5 marks)

Find $f'(x)$ of the following function using the Delta Method (no marks will be assigned if you use other methods of differentiation):

$$f(x) = 2x^2 - 5x + 1.$$

Problem 3.(6 marks) Find y' by implicit differentiation:

$$y^3 + 2yx^2 + x^2 - 3y^2 = 0$$

and find the slope of the tangent line to the curve at the point $(0; 3)$.

Problem 4.(4+4+4+4 marks) Find the derivatives of the following functions using any method (no need to simplify):

$$a)\quad f(x) = (x^2 + 2)^3(x^3 + x - 1)$$

$$b)\quad f(x) = \frac{(x^2 - 1)}{(x^2 + 1)^{1=2}}$$

c) $x^2 \cot(3x + 1)$

d) $2 \arctan(x^4)$

Problem 5.(5+2 marks) Consider the function

$$f(x) = \frac{1}{\sqrt{x^2 + 9}}$$

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a) Find the equation of the tangent line to the graph of $f(x)$ at $x = 4$.

b) Find the equation of the normal line to the grapho of $f(x)$ at $x = 4$.

Problem 7.(5+5 marks) Consider the function

$$f(x) = 6x^3 - 9x^2 + 1;$$

Problem 8.(4 marks) The temperature in degrees Celsius of an object at time t (in minutes) is: $T(t) = \frac{3}{8}t^2 - 15t + 180$ for $0 \le t \le 20$. A what rate is the object cooling when $t = 10$?

Problem 9.(5 marks) A 5.00 m ladder is placed against a wall. If the bottom of the ladder slides away from the wall at 0.200 m/s, how fast is the top of the ladder moving when it reaches 2.5 m above the ground?

Problem 10.(5 marks) A box without lid is being produced by cutting out 4 squares from the corners of a rectangular piece of metal of size $30cm$ by $40cm$ and folding up the sides so obtained. Find the maximal possible volume of the box.

Problem 11.(5+5 marks) A partcile is moving along a straight trajectory according to the law: $s(t) = t^3 - 3t + 1$, $t \geq 0$, where s is in meters and t is in seconds.

a) Find the times at which the particle is stationary and its acceleration at those times.

b) Find the intervals of time in which the particle is moving in the positive direction.

Problem 12.(5+5 marks) Evaluate the following integrals:

$$a) \quad \int 2x^{5/3} - \frac{2}{x^2} - 1 \, dx =$$

$$b) \quad \int \frac{3x}{x^2 + 1} \, dx =$$

Problem 13.(5 marks)
Find the function $f(x)$ that satisfies: $f'(x) = 3x^2 + 2x + 3$ and $f(0) = 1$.

Answers:

1. a: 10/3; b: -3/5; c: 5/2

2. $4x - 5$

3. $y^0 = \frac{-4xy-2x}{3y^2+2x^2-6y}$, $m = 0$

4. a: $6x(x^2+2)^2(x^3+x-1)+(x^2+2)^3(3x^2+1)$; b: $\frac{2x(x^2+1)^{1=2}-x(x^2-1)(x^2+1)^{-1=2}}{x^2+1}$; c: $2x\cot(3x+1) - 3x^2\csc^2(3x+1)$; d: $\frac{8x^3}{1+x^8}$

5. a: $y = \frac{-4}{125}x + \frac{42}{125}$; b: $y = \frac{125}{4}x - \frac{624}{5}$

6. -4π

7. a: increasing in $(-1;0)$ $[$ $(1;+1)$, decreasing in $(0;1)$; rel. max at $x = 0$, rel. min. at $x = 1$, b: concave down in $(-1;1=2)$, concave up in $(1=2;+1)$; in ection point at $(1=2;-1=2)$.

8. $-7:5\,^o{=}min$

9. $-0:346\,m{=}s$

10. $V_{max} = 3032:30\,cm^3$

11. a: $t = 1$, $a(1) = 6\,m{=}s$; b: $(1;+1)$

12. a: $\frac{3}{4}x^{8=3} + \frac{2}{x} - x + C$; b: $\frac{3}{2}\ln jx^2+1j + C$

13. $\overset{2}{f}$